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COMMENT

Random surfaces on hard spheres†

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Abstract. The relation between mass, M, and size, R, for self-avoiding random surfaces enclosing a hard sphere of radius r is examined. It is suggested that $M \sim R^2F[(R-r)/r]$, with F proportional to the surface roughness. These functions are investigated using experimental data and theoretical arguments.

Recently, there has been great theoretical interest in the study of the properties of self-avoiding random surfaces [1, 2]. It seems to have been established from experiments [3] and computer simulations [4] that the average size or radius of gyration R of these surfaces scales with the linear size L of the manifold as $R \sim L^n$, with $n = 0.8 \pm 0.05$. Then, the unfolded area or the mass M of these objects scales with the radius as $M \sim L^2 \sim R^{2/n} = R^D$, where the fractal dimension D assumes the value $D = 2.50$ with typical fluctuations of 6%. These numerical results for D are in agreement with a Flory-type argument which predicts $D_{\text{Flory}} = \frac{5}{2}$ [2].

In this comment we examine the behaviour of self-avoiding random surfaces on hard spheres for the first time. We started with 938 square sheets of paper of different edge L and proceed to crumple them in such a way as to enclose completely rigid spheres of radii $r_1 = 0.60$ cm, $r_2 = 0.85$ cm, $r_3 = 1.28$ cm, $r_4 = 2.71$ cm and $r_5 = 3.33$ cm. The ensemble of random surfaces was formed by five groups, comprising: groups 1 and 2 both with 175 crumpled balls (divided into $f_1 = f_2 = 25$ families of seven balls each) involving, respectively, spheres of radii r_1 and r_2 and sizes $L = 6$, 10, 15, 20, 30, 45 and 66 cm; group 3 with 294 crumped surfaces (divided in $f_3 = 49$ families of six balls each) involving spheres of radius r_3 and sizes $L = 10$, 15, 20, 30, 45 and 66 cm; and finally groups 4 and 5, both formed from $f_4 = f_5 = 49$ families of three balls each involving, respectively, spheres of radii r_4 and r_5. For the last two groups L is limited to $L = 35$, 50 and 66 cm as a consequence of the maximum size of the paper sheets normally available (66×96 cm) and the relatively large magnitude of r.

From the theoretical point of view it is reasonable to assume that $M \sim R^2F[(R-r)/r]$, where M is the mass or area of the manifold, $R > r$ is the radius of gyration, $0.1 \leq [(R-r)/r] \leq 6$ in our experiment, and $F(x)$ is a scaling function with the following properties:

(i) $F(x) \simeq 1$, for $x \ll 1$, i.e. $M \sim R^2$ (which is the mass-size relation for the unfolded Euclidean covering of the hard core);

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(ii) $F(x) \sim R^{D_0-2}$, for $x \gg 1$, i.e. $M \sim R^{D_0}$, where D_0 is the mass-size exponent for random surfaces without a hard core boundary condition. As previously stated D_0 is typically 2.5 ± 0.15 [3, 4].

From the experimental data we calculated the surface roughness σ for these surfaces. After picking out at random from the ensemble $\sqrt{f_i}$ surfaces associated with a given r and L, we measured for each one of these the external radius R_k along five directions taken equally at random. σ is defined as the square root of the variance $(1/N)\sum_{k=1}^{N}(R_k - R)^2$ which is an average over a large number $N = 5\sqrt{f_i} \geqslant 25$ of observations of the surface topography. We found that $\sigma(\exp)_i \sim [(R - r_i)/r_i]^{\alpha_i}$, with $\alpha_1 = 0.56 \pm 0.07$, $\alpha_2 = 0.6 \pm 0.08$, $\alpha_3 = 0.64 \pm 0.11$, and $\alpha_4 = \alpha_5 = 0.7 \pm 0.2$, where the subscripts refer to groups 1 to 5. It can be shown [5] that the surface roughness for a fractal surface of dimension D and maximum extent L scales as $\sigma \sim L^{(3-D)}$ in the physical space. Then, for a random surface of radius R, $\sigma \sim R^{(D/2)(3-D)}$ since in this case $L^2 \sim R^D$. We hypothesise in conformity with $\sigma(\exp)$ that for random surfaces involving hard cores, σ scales with $(R-r)/r$ as $\sigma \sim [(R-r)/r]^{(D/2)(3-D)}$. The fractal dimension D_i for the random surfaces of the group i may be estimated if the relation $\alpha_i = (D_i/2)(3 - D_i)$ is adopted. The values obtained are then $D_1 = 2.56 \pm 0.07$, $D_2 = 2.52 \pm 0.08$, $D_3 = 2.48 \pm 0.11$ and $D_4 = D_5 = 2.4 \pm 0.22$. These values are physically consistent among themselves (in the sense that D increases when r decreases) and D_1,

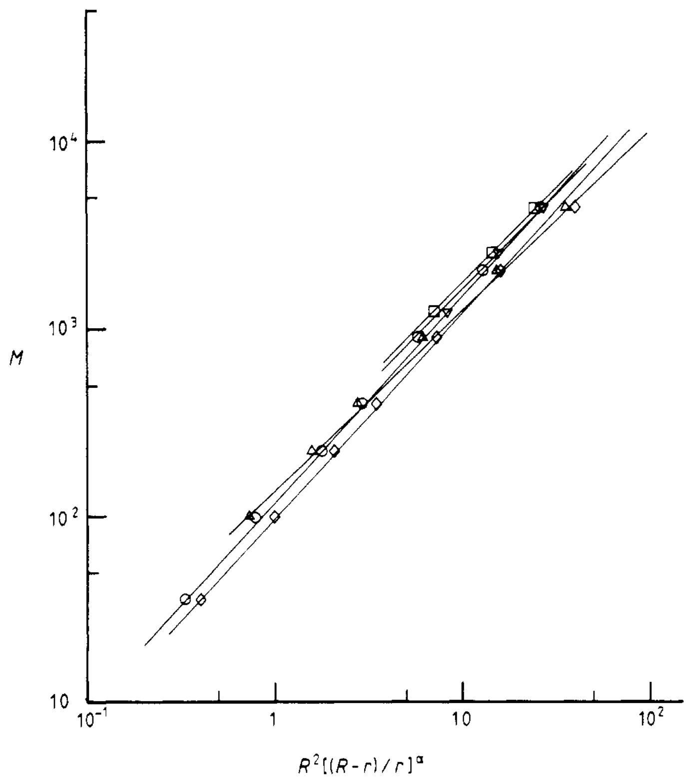

Figure 1. Log-log plot of the mass as a function of $R^2[(R - r_i)/r_i]^{\alpha_i}$, $\alpha_i = (D_i/2)(3 - D_i)$, as obtained from the experimental data: $r_1 = 0{\cdot}6$ cm (◇); $r_2 = 0.85$ cm (○); $r_3 = 1.28$ cm (△); $r_4 = 2.71$ cm (□); $r_5 = 3.33$ cm (▽). The straight lines have slopes equal to the unit with a confidence limit of 95%.

which corresponds to the smallest value of r, is close to the value D_0 observed for self-avoiding random surfaces without hard cores [3, 4]. Furthermore, the experimental data show a linear scaling of M with $R^2[(R-r)/r]^\alpha$ for all groups of surfaces studied (figure 1).

The previous results suggest that the function $F(x)$ probably behaves as $F \sim x^{(D/2)(3-D)}$. Finally, we can check the last scaling relation by using the condition (ii) above as a self-consistent condition, namely that $F \sim [(R-r)/r]^{(D_0/2)(3-D_0)} \sim R^{D_0-2}$, for $[(R-r)/r] \gg 1$. The solution for the exponent D_0 gives $D_0 = 2.5615\ldots$ which is, in fact, in good agreement with previous works [3, 4].

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